8/28/25 # Lecture 2

Logic

propositions

1. It is raining now PROPOSITION
2. Today's weather is good PROPOSITION
3. Taeho is handsome PROPOSITION

$\oplus$ = exclusive or

Conditional Statements

- "if ~ then"
- $p \to q$ "if p then q"
  - p is the hypothesis
  - q is the conclusion
- The only time a conditional statement is false is when the hypothesis is true but the conclusion is false.

Variations of conditional Statements (DON'T NEED TO KNOW)

- converse: flip
- inverse: negate both
- contrapositive: flip and negate

ICE #02

- The conditional proposition and its converse have the same meaning } False
- The conditional proposition and its inverse have the same meaning } False
- The converse and inverse of a conditional proposition have the same meaning. } True
- The conditional proposition and its contrapositive have the same meaning. } True

Biconditional Statements

- "if and only if"
- $p \leftrightarrow q$
- only true when p and q have the same truth value

Logical Equivalence

- Tautology: coumpound proposition that is always true
- Contradiction: compound proposition that is always false
- Logically Equivalent: Two statements are logically equivalent if they have the exact same truth values.
- De Morgan's Laws: These laws show that the negation of a conjunction is the disjunction of the negations and vice versa.

$\boxed{Ex}$ $\neg(p \wedge q \vee r) \equiv (\neg p) \vee (\neg q) \wedge (\neg r)$

$$\left(\bigwedge_{i=1}^{n} a_i\right) = (a_1 \wedge a_2 \wedge a_3 \ldots \wedge a_n)$$

$$\neg\left(\bigwedge_{i=1}^{n} a_i\right) = (a_1 \vee a_2 \vee \ldots \vee a_n)$$ ← by DeMorgans

Important Later To Memorize

$p \rightarrow q \equiv \neg p \vee q$

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

Proof Example

$\neg(\neg(\neg p \wedge q) \wedge (p \vee q))$	
$\neg\neg(\neg p \wedge q) \vee \neg(p \vee q)$	De Morgan Law
$(\neg p \wedge q) \vee \neg(p \vee q)$	Double Negation
$(\neg p \wedge q) \vee (\neg p \wedge \neg q)$	De Morgans Law
$\neg p \wedge (q \vee \neg p)$	Reverse of Dist Law
$\neg p \wedge T$	Complement
$\neg p$	Identity